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INTERNAL RATE OF RETURN TO INVESTMENT

PROJECTS WITH FUZZY CASH FLOWS

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ABSTRACT

- *The more important techniques of discounted cash flow, as the net present value and internal rate of return can be extended to investment projects with fuzzy cash flows*
- *This paper extends the concept of net present value for trapezoidal fuzzy numbers applied to independent or mutually exclusive projects with different selection criteria.*
- *Also, it defines the trapezoidal and triangular rates of return and other variants the average and intermediate rates of return and criteria applied in each case.*
- *All these techniques can be applied relatively easy in a spreadsheet or using specialized software.*

INTRODUCTION

- *The net present value and the internal rate of return can be extended to investment projects with fuzzy cash flows (Chiu and Park, 1994), without some properties of real models, mainly because the algebraic structure of fuzzy numbers exclude the distributive propety or multiplicative inverses (Dubois, 1979; Buckley, 1987; Ríos, 1999).*
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- *The applications of these techniques to the investment decisions are numerous in Capital Budgeting (Kahraman et al., 2002) and Portfolio Selection (Huang, 2007).*

TRAPEZOIDAL RATE OF RETURN

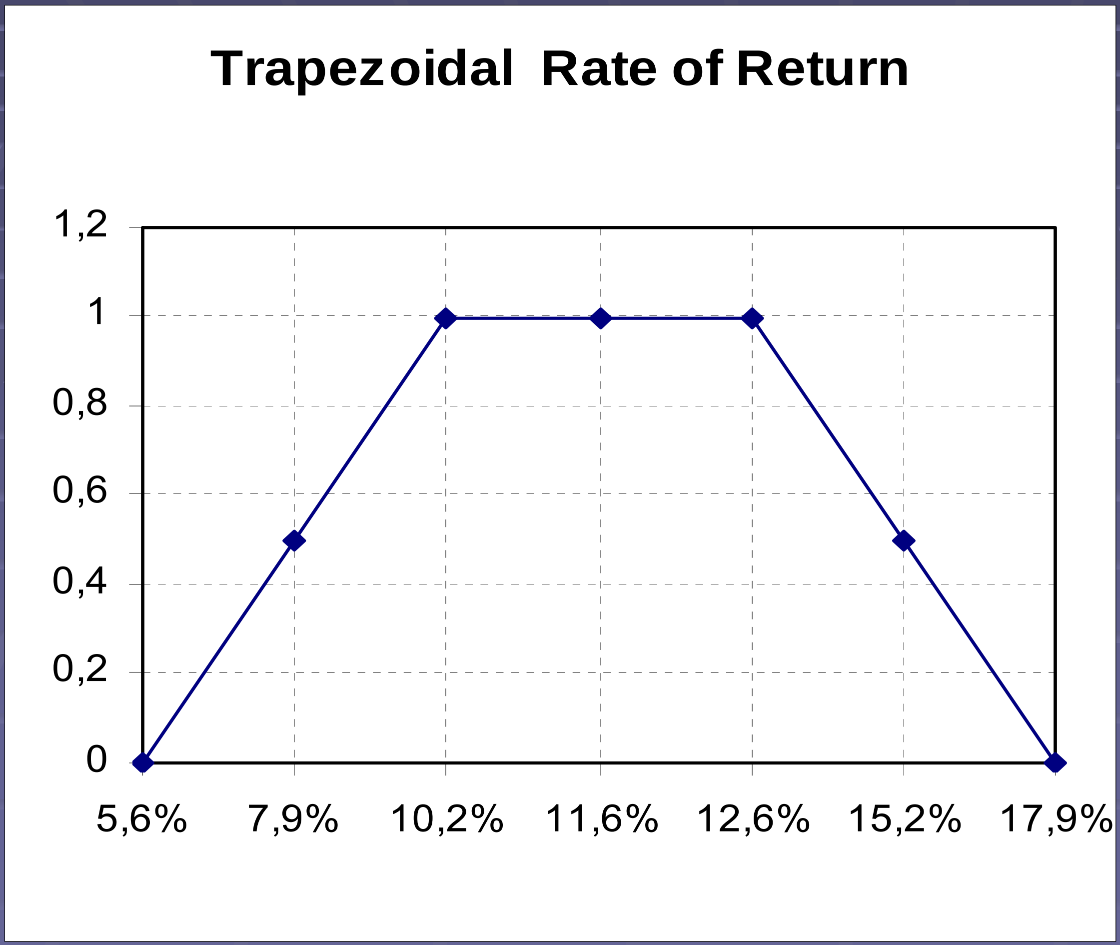
- *A trapezoidal fuzzy cash flow*
  Fj = [Fj1, Fj2, Fj3, Fj4] is conventional
  if it satisfies the following conditions:
  $Foi < 0$, $Fji > 0$, $\sum_{j=0..n} Fji > abs(Foi)$
  $i = 1,2,3,4$ *and* $j = 1,2,...n$.

- *Under these conditions exists for each i a single real internal rate Ri> 0 such that*

  $$\sum_{j=0..n} Fji*(1+Ri)^{-j} = 0.$$

- *Moreover, since by hypothesis*
  $Fj1 \leq Fj2 \leq Fj3 \leq Fj4$
  also have to $R1 \leq R2 \leq R3 \leq R4$

TRAPEZOIDAL RATE OF RETURN



TRR	alfa
5,6%	0
7,9%	0,5
10,2%	1
11,6%	1
12,6%	1
15,2%	0,5
17,9%	0

Application Criteria

- *Let Fj = [FJ1, Fj2, Fj3, FJ4] a conventional fuzzy cash flow and K = [K1, K2, K3, K4] the discount rate.*

  If FIRR > K then NPVD > 0

  if FIRR < K then NPVD <0.

- *As a corollary of the above result there is a criterion for selecting the FIRR for conventional fuzzy cash flows consistent with the criterion of NPVD: If FIRR > K then NPVD > 0*

  and therefore, the project is accepted.

  If FIRR < K then NPVD < 0

  and therefore the project is rejected.

TRIANGULAR RATE OF RETURN

- Given the FIRR = [R1 , R2 , R3 , R4], it is possible to construct a triangular rate of return TRR = [Rm , Rp, RM], defining Rp for cut α=1 as the geometric mean of R2 and R3 ,

$$(1+Rp)^2 = (1+R2)(1+R3)$$

$$(1+Rm)^2 = (1+R1)(1+R2$$

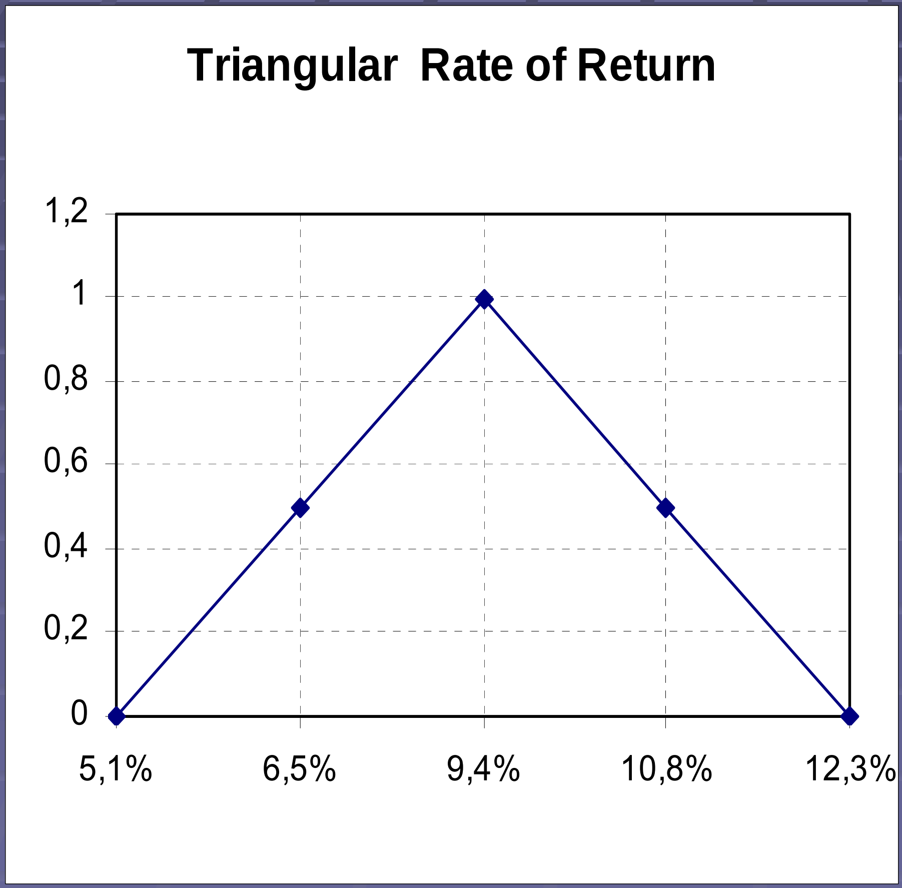
$$(1+RM)^2 = (1+R3)(1+R4).$$

The NPVD discounted at the

TRR is a triangular fuzzy number

contains a zero for some α-cut.

TRIANGULAR RATE OF RETURN



TRR	alfa
5,1%	0
6,5%	0,5
9,4%	1
10,8%	0,5
12,3%	0

Intermediate Rate of Return

- For conventional cash flow **Fj**,

the Intermediate Rate of Return

InRR = $[R_a , R_b]$ is defined by equations

$$\sum_{j=0..n} (Fj1 + Fj2)(1+R_a)^{-j} = 0$$

$$\sum_{j=0..n} (Fj3 + Fj4)(1+R_b)^{-j} = 0.$$

Both R_a and R_b exist because the sum of conventional cash flow is conventional, and it follows that $R1 < R_a < R2$ and $R3 < R_b < R4$.

Application Criteria

- Let Fj j = 1 ... n, a conventional fuzzy cash flow, **K** the fuzzy minimun rate and E(**NPVD**) the mean value of fuzzy net present value

(Dubois-Prade)

If **InRR > K** then E(**NPVD**) **> 0** and

if **InRR < K** then E(**VPND**) **< 0**

Average Rate of Return

- Given a conventional cash flow

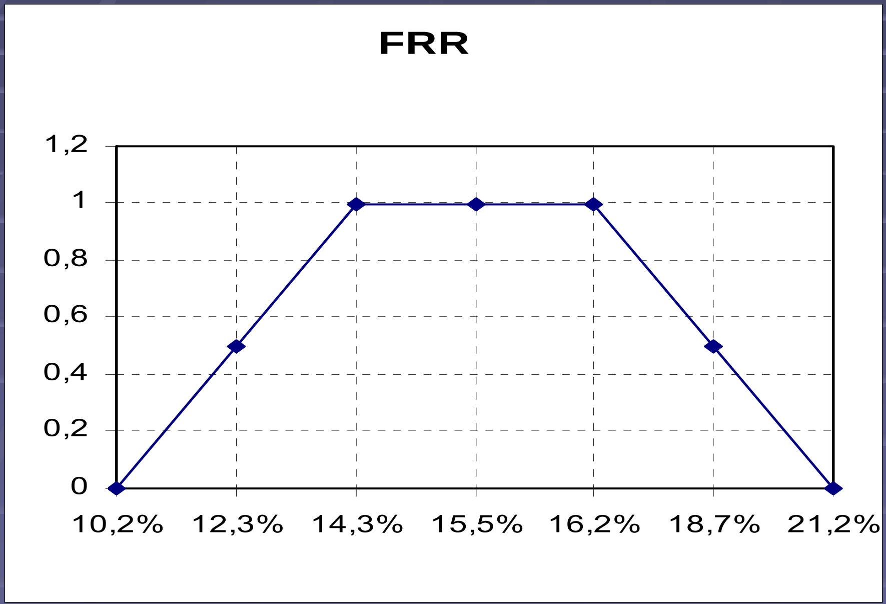
Fj = [F_{j1} , F_{j2} , F_{j3} , F_{j4}] , we can determine a real number R_λ , $0 \leq \lambda \leq 1$, with the folowing property

$$(1-\lambda)\Sigma_{j=0..n} (F_{j1}+F_{j2})(1+R_\lambda)^{-j} +$$

$$\lambda \Sigma_{j=0..n} (F_{j3}+F_{j4})(1+R_\lambda)^{-j} = 0$$

- The average rate R_λ exists because the sum of conventional cash flow is conventional

Intermediate and Average Rate of Return



FRR	alfa
10,2%	0
12,3%	0,5
14,3%	1
15,5%	1
16,2%	1
18,7%	0,5
21,2%	0

Mean Value (Dubois/Prade)

12,3%	18,7%

Average Value(λ = 0.5)

15,5%

Application Criteria

- Let **Fj** $_{j=1...n}$, be a fuzzy cash flow, **K** the fuzzy minimum rate and V_λ(**NPVD**) the average value of the fuzzy net present value for a fixed value of λ.

- If $R_\lambda > K4$ then V_λ(**NPVD**) > 0 and if $R_\lambda < K1$ then V_λ(**NPVD**) < 0.

CONCLUSIONS

- The construction of the cash flow of an investment project involves to estimate future revenues and expenditures, which depend on other parameters such as interest rates, inflation or exchange rate.
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- These variables are generally uncertain, and if there is not reliable probabilistic information, you can define fuzzy cash flows that represent reasonably available information.

CONCLUSIONS

- In this research, we define the trapezoidal and triangular rates of return and other variants applied at intervals as the average and the intermediate rate of return and the criteria applied in each case

- All these techniques can be applied easily in a spreadsheet or using specialized software.