

Option to Renew in Leasing of Companies

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Abstract

The techniques of discounted cash flow are insufficient to assess the diversity of options that can be associated with the leases. For this reason, the explicit evaluation of these options is critical to complete an analysis of the lease. One of the most relevant researches on valuation of real options in leases was proposed by Grenadier (2005) who develops a unified model to analyze different types of options. The model is sufficiently flexible to determine the equilibrium rent with various structures and options.

Moreover, the leasing of a company is a contract by which the lessee agrees to pay a series of annual dues to the landlord for a certain time, in exchange for the use of the company as their own (Rojo, 1998). The main objective of this paper is to determine the value of the option to renew the lease by the lessee and its effect on the rent under the assumption that the acceptable maximum annual rent for the lessee is the net cash flow generated by the company. The value of the option to renew can be obtained by the Black's model of options on the futures.

Introduction

The options approach has been widely applied to the valuation of mortgages (Brennan and Schwartz, 1983; Childs et al., 1996) and the condo market (Grenadier, 1995). In addition, various options associated with the leases as renew, cancel or adjust income have been studied by different authors (Booth and Walsh, 2001; Adams et al, 2001; Baum, 2003). On the other hand, Grenadier (1995) provides a unified approach to determine the value of the lease and structure of rents in real standard lease contracts. The model is very flexible to determine equilibrium rent with various structures and options. In particular, it explores options to cancel or renew a lease and the value of the insurance premium for the lessor. In a later work, Grenadier (2005) applied their model to leasing real estate such as offices, shops or industrial properties that represent a significant amount of the rental market model. The approach is strictly economic and it is based on the principle that leasing an asset is equivalent to buying the use of the asset for a period of time. Thus, the lease provides a mechanism that allows the separation of ownership and use. While the tenant benefits from the use of the asset, the lessor receives rent payments plus the residual value of the asset. This simple and intuitive model facilitates the analysis of complex leases but does not consider important aspects like taxes in the decision to buy or lease (Schallheim, 1994, Gordon, 2001).

Furthermore, Hendershott and Ward (2002) study the leases in shopping centers that generally have clauses with the option of renewal for the lessee and the option to adjust the rent depending on sales volume. They also study the case of the dual option, that is to say, the contract contains two options, renewal for the lessee and adjusting rent for the landlord. Additionally, it may include other options such as canceling the contract by both parties or the option for the tenant to sublet. The basic principle for determining the rent of option contracts is the same as used by other authors (Grenadier, 1995; McConnell and Schallheim, 1983), the lease with the option to adjust the rent must be less than the rent of the basic contract if the other conditions remain the same. Similarly, the contract with the option to renew should have a higher rent. In the case of the dual option, you can show the rent is equal to the base rent.

Another important work in this field, is developed by Bellalah (2002), which presents a unified framework for assessing leases under the real options approach with incomplete information model. The model is similar to the traditional models of term structure of interest rates. It can be applied to different leases with options to renew or cancel the lease. Finally, Buetow and Albert (1998) study the option to renew the lease on a property when the exercise price is a fraction of the market price. Their results are important, but the numerical methods used are complex to implement in practice. Clapham (2003) extends the work of these authors by means of analytical solutions and compared with the results obtained by Grenadier (2005).

Options on Futures: Black's Model

Black's model (1976) can be used to value options on futures contracts and is based on the assumption that the future price F follows a geometric Brownian motion

$$dF = \mu F dt + \sigma F dz$$

where μ is the expected growth rate, σ the volatility and dz a Wiener process. If the risk-free rate r is considered constant, one can obtain closed formulas for European options on futures. In the case of the European call option (C) with strike price X and maturity t , the solution obtained by Black is (Hull, 2000)

$$C = e^{-rt} (FN(d_1) - XN(d_2)) \quad \text{where} \quad d_1 = \frac{\left[\ln\left(\frac{F}{X}\right) + \left(\frac{\sigma^2}{2}\right)t \right]}{\sigma\sqrt{t}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{t}$$

Black's model assumes that the volatility of F is constant, but this assumption can be relaxed. The values of underlying asset V and future F before maturity T are irrelevant because options are European. In addition, F is the expected value of V_T in the risk neutral world and sufficient conditions for using the Black model are: V_T has a log normal distribution, $\sigma\sqrt{T}$ is the deviation of $\ln V_T$, interest rates are not stochastic

In short, Black's model is very flexible and can be used to assess a wide range of European options when interest rates are not stochastic and the underlying variable V has a lognormal distribution to option expiration. This implies that it is not necessary to assume that V or F follows a geometric Brownian motion and thus, strictly speaking, σ is not volatility. We simply say that $\sigma\sqrt{T}$ is the standard deviation of the variable $\ln V_T$ and σ is a measure of the volatility of V at maturity T (Hull, 2000)

Value of the renewal option

The main objective of this paper is to determine the value of the option to renew the lease by the lessee and its effect on the rent under the assumption that the acceptable maximum annual rent for the lessee is the net cash flow generated by the company (Rojo, 1998). If a lease for n years, gives the lessee the option to renew by m years at the end of year n , and CR_j is the new lease fee $j=n+1 \dots n+m$, then the tenant renewal option is a European call option on the future value F ,

$$F = \sum_{j=n+1}^{n+m} \frac{CM_j}{(1+k_o)^j} \quad \text{and} \quad \text{exercise price} \quad X = \sum_{j=n+1}^{n+m} \frac{CR_j}{(1+k_o)^j}$$

where CM_j is the acceptable maximum fee equal to the lessor's net cash flow available, generated by the company leased. The discount rate is the weighted average cost capital k_o . Thus, the value of the option to renew OR can be obtained by the model Black

$$OR = e^{-rt} (FN(d_1) - XN(d_2))$$

If rent CR_j is a fixed proportion b of the maximum quota CM_j then the exercise price is

$$X = \sum_{j=n+1}^{n+m} \frac{CR_j}{(1+k_o)^j} = \sum_{j=n+1}^{n+m} \frac{b * CM_j}{(1+k_o)^j} = bF$$

In this case, the value of the option to renew can be expressed as a function of factor b

$$OR = e^{-rt} F(N(d_1) - bN(d_2))$$

Lease fee with Renewal Option

If CAR_j is the lease fee in year j , $j = 1 \dots n$, when the lease gives the lessee the option to renew by m years at the end of year n , and CA_j is the lease fee in the same period without renewal option, then the present value of the lease with the option to renew must be equal to the present value of the lease without an option to renew plus the value of the option to renew OR (Grenadier, 2005), that is to say

$$\sum_{j=1}^n \frac{CAR_j}{(1+k_o)^j} = \sum_{j=1}^n \frac{CA_j}{(1+k_o)^j} + OR$$

If the lease fee is proportional to the maximum fee CM_j , that is to say, $CA_j = b1 \cdot CM_j$ and $CAR_j = b2 \cdot CM_j$ then the equation becomes

$$\sum_{j=1}^n b2 \cdot \frac{CM_j}{(1+k_o)^j} = \sum_{j=1}^n b1 \cdot \frac{CM_j}{(1+k_o)^j} + OR$$

Hence it follows that

$$b2 - b1 = \frac{OR}{\sum_{j=1}^n \frac{CM_j}{(1+k_o)^j}}$$

In this way, we obtain

$$b2 - b1 = \frac{e^{-rn} F(N(d1) - bN(d2))}{P}$$

$$\text{where } P = \sum_{j=1}^n \frac{CM_j}{(1+k_o)^j} \quad \text{and} \quad F = \sum_{j=n+1}^{n+m} \frac{CM_j}{(1+k_o)^j}$$

If we establish a rate implicit equivalence r_n , such that $F = P \cdot (1+r_n)^n$, then

$$b2 - b1 = e^{-rn} (N(d1) - bN(d2)) \cdot (1+r_n)^n \quad (3.37)$$

This means that the rate implicit equivalence r_n can be used as a parameter to estimate the increase in the rent when the lessee has the option to renew.

Application to leasing of a company

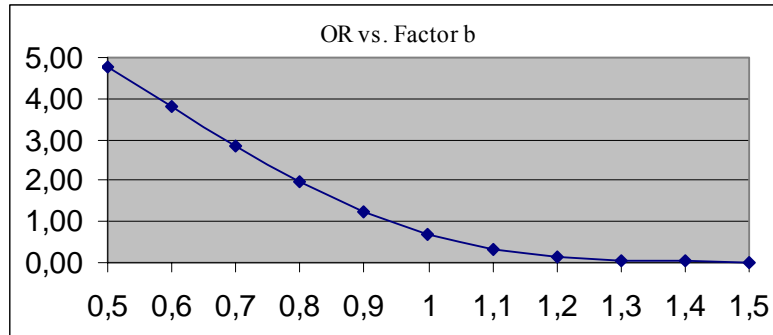
In this example, it is shown the case of a company leased with renewal option. The lease is for 5 years with an option to renew for another 5 years (Ríos, 2009, p.259). The value of the option to renew OR is obtained by different values of the factor b in the following table

Table 1 Renewal Option Parameters

r	n	VA5(CMj)	σ	ko	i	VA(CMj)	rn
4%	5	11,602882	8%	15%	6%	5,806037	14,852%
b	d1	d2	N(d1)	N(d2)	OR	b2-b1	CAR-CA
0,5	3,964253254	3,785367816	0,999963187	0,999923259	4,749832844	0,81808515	1,416949
0,6	2,945044768	2,766159329	0,99838546	0,997163959	3,800681713	0,65460856	1,133802
0,7	2,08331627	1,904430832	0,981388798	0,971572952	2,862123812	0,4929565	0,853816
0,8	1,336853093	1,157967655	0,909364703	0,876561382	1,97702244	0,3405115	0,589777
0,9	0,678425907	0,499540469	0,751249156	0,691300658	1,22619939	0,21119385	0,365794
1	0,089442719	-0,089442719	0,535634963	0,464365037	0,677038356	0,11660937	0,201971
1,1	-0,443357383	-0,622242822	0,328753628	0,266891106	0,334134605	0,05754951	0,099678
1,2	-0,929765768	-1,108651206	0,176246187	0,133790339	0,149123194	0,02568416	0,044486
1,3	-1,377218106	-1,556103545	0,084222413	0,059841736	0,06106514	0,01051752	0,018217
1,4	-1,791494265	-1,970379703	0,036607007	0,024397435	0,02327979	0,00400958	0,006945
1,5	-2,177176141	-2,35606158	0,014733714	0,009234928	0,00837224	0,00144199	0,002498

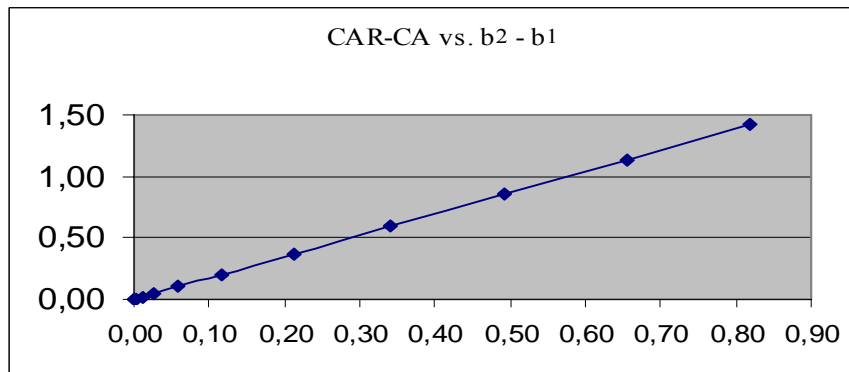
It is observed that the value of the option to renew is a decreasing function of the factor b , therefore, the exercise price. For very low values of the exercise price, the option value is very high and this will result in a further increase in the lease fee, which favors the landlord.

Graphic 1 Renewal Options vs. Factor b



The Graphic 2 shows the increase in the rent when the lessee has the option to renew.

Graphic 2 Increase in the rent vs. differential b_2-b_1



The increase in lease fee is an increasing linear function from the differential in rent factor. In this case, the rate implicit equivalence to estimate the increase in the rent when the lessee has the option to renew is 14.85%

Conclusions

In this work, the leasing of a company is a contract by which the lessee agrees to pay a series of annual dues to the landlord for a certain time, in exchange for the use of the company as their own. We study the value of the option to renew the lease by the lessee and its effect on the rent under the assumption that the acceptable maximum annual rent for the lessee is the net cash flow generated by the company. The Black's model for options on futures is used to determine the value of the option to renew the contract by the lessee, considering the residual value of the firm as the value of the company at the end of the lease period.

On the other hand, the equilibrium principle of Grenadier is applied to determine the rent of lease. This model is a strictly economic approach applicable to a variety of options in leases. Finally, a sensitivity analysis of the option value and the lease fee is done in a particular case. The methodology developed in this work is applicable to different types of options associated with the leases.

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