A Search for Patterns in the Rigidity Matrix of Plane Structures.

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INTRODUCTION

Teaching matrix analysis methods has slowly supplanted traditional structural teaching, where many methods were taught according to the structural types we were dealing with. But, apparently, it fails to establish immediate relationships with the traditional methods of structural analysis. This is difficult for the students, which do not manage to relate what is left of the traditional approach with the modern methods. Even algorithmic programming, which was the original target of matrix analysis, is now almost out of place, because the normal engineer cannot compete with the capabilities of the software companies. They provide complete self contained packages which makes the engineer think that all he needs to do is press keys and feed data, even knowing nothing about the innards of the software he uses, leading many times to real blunders. In other words, it is like speaking a language without being able to think in that language.

Add then the constant pressure we are receiving from our universities to reduce the number of hours dedicated to structural teaching. This tendency arises to make room for other subjects like humanities, management, accounting and so many other "modern" subjects. Most of them tend to give preference to operational skills or the creation of attitudes instead of a solid basic knowledge, the whole process ending up with the effective devaluation of structural engineering. In the case of the author of this paper, he went through a reduction of the "professional cycle" of structural teaching from five subjects to just two, in a period of only four years. There was no way out but trying to condense several seemingly unrelated subjects, such as Dimensional Analysis, Graph theory (Pahl 2001, Trudeau 1993), Linear Algebra (Strang 1998) and Matrix Analysis (West 1993); (Sennett 2000, Uribe 2000, McGuire 2000) and Cross and Slope-deflection equations (Belluzzi 1956) into a single course (Cubides 2003, Villalobos 2003), to make room for the lost subject matter through a simplification process based on synthesizing. This paper will try to explain this process, which has been, in practice, successful with the student body of the two universities where is has been taught.

Keywords: **Structures, Matrix Analysis. Teaching of Structures.**

QUESTIONS ASKED: a) WHY GRAPH THEORY? b) WHY DIMENSIONAL ANALYSIS? c) WHY CROSS? d) WAS MATRIX STRUCTURAL ANALYSIS ALREADY INVENTED WHEN IT STARTED WITH THAT NAME?

a) Perhaps, for undergraduate students, the most difficult aspect of Matrix Analysis, lies in the sudden change from structures analysed inside the Euclidean Geometry, with determined forms, orientations and metrics, and then jump into a topological space, where only connectivities matter.

The second one is the abandoning of forces as unknowns, and taking deformations as such.

The third is to mentally absorb the idea that the deformations we are dealing with are relative, not absolute, as it happens when one usually deals with topographical measurements.

The fourth is the apparently magical character of the idea of not having to think at all about the pertinence of a certain variable, because almost everything is automatic.

All this becomes perfectly clear when one **appeals to graph theory, (which is nothing more than the visual representation of the algebra of relations** (or connectivities), all expressible with matrices. The author has not found yet a single structural analysis textbook using this approach, which is rather curious.

b) When looking at many textbooks of matrix analysis, one notices that many times matrices do not follow a fixed order, they may be handled freely in different ways (Laredo 1970) and the old Fourier Principle of Dimensional Homogeneity for physical equations is not taken into account explicitly.

If one consistently applies Dimensional Analysis, the idea of isomorphisms becomes, in fact, clear for different relations between forces and displacements, then the evident emerging patterns of regularity help to understand how the same system looks like from different points of view.

Besides, Structural Matrices have dimensions, they cannot be handled as purely numerical entities, as it is done on most textbooks, which ignore or put aside this fact.

c) Why the Cross-Method: first, on learning it, meanings and processes become intuitive; second, if one observes the typical Rigidity Matrices shown on textbooks, the amount of information needed can be cut by 75% by making use of the concept of Carry-0ver Factors. Which in reality is the same thing as transposing a matrix because Rigidity Matrices can be divided into four quarters related either by symmetry or by proportionality, or by other functional relations (trigonometric f. i.)

d) Apparently Matrix Methods were already implemented, with another name, when Takabeya published in 1930 his method for systematically solving the Slope-Deflection equations, (Belluzzi, 1956). He was very clever by adopting a totally non-dimensional matrix, based on a single type of rigidity coefficient (EI/L) and using only angles (both node rotations and floor drifts) as non-dimensional unknowns. Takabeya's procedure was apparently ignored in the U.S.A. because he published his paper in German, and so, it became a European method only. His method made use of Graph theory, dimensional analysis and used PATTERNS instead of procedures to order the rigidity Matrix Terms. He even went to extremes by trying to "adopt" similar orders of magnitude for the coefficients in order to improve the precision of the results, this procedure tended to obscure the underlying simple methodology, from the point of view of losing track of the coefficients formal contents.

What the Simplified Matrix does to simplify work:

The Simplified Matrix Analysis does this: 1) It uses global coordinates only. 2) Except for the final step of multiplying the flexibility matrix by the load vector, no intermediate matrix calculations need to be performed. 3) The "finite element" is no more every lone member, but each structural node together with all its concurring members. 4) The rigidity matrix is divided into nine submatrices, each submatrix being dimensionally homogeneous. 5) All submatrices are isomorphic with the adjacency matrix (node-node) used to describe the structure using Graph Theory. 6) All the necessary numerical data to perform the structural analysis is contained in a few tables that can be superimposed, to multiply numbers cell by cell. 7) The Matrices can be easily filled by hand. 8) Instead of suppressing degrees of freedom, each member is described Cross-style, using, if necessary, modified rigidities and modified carry-over factors. 9) All plane structures representable by closed networks can be solved this way. 10) Some special cases, like external hinges receiving two members, or inclined rollers as external supports can be solved with some additional steps. 11) The student goes immediately from the Cross-Method to Matrix analysis having to learn only a limited set of rules. 12) The sums of rigidities are performed row by row to diagonal cells without recurring to programs. It can be performed easily with EXCEL™ or similarly popular programs.

Table 1 shows the Rigidity Matrix contents and Architecture for each member end exiting from an active node*. Each one of the nine submatrices contains terms of the same type as the ones shown in this table.*

*Members are identified by repeating systematically the coordinate pairs node-node in each submatrix. There are similar tables for carry-over factors and for special member-ends types. With these arrangements it is easy to remember the fixed patterns of the isomorphic submatrices****. The writer is of the opinion that these procedures give preference to structural engineering teaching over computing methods.***

Table 1 below corresponds to all members being built-in into their end nodes but It can be easily modified for other member ends conditions (either end or both)

TABLE 1. NODAL RIGIDITY MATRIX

	Rotation □	Displacement □x	Displacement □y
MOMENT M	4EI/L	(6EI/L^2) sin□□	-(6EI/L^2) cos□□
FORCE X	(6EI/L^2) sin□□	(AE/L)cos^2□□□(12 EI/L^3)sin^2□	[AE/L- 12EI/L^3] sin□cos□
FORCE Y	-(6EI/L^2) cos□□	[AE/L- 12EI/L^3] sin□cos□	(AE/L)sin^2□□□(12E I/L^3)cos^2□

The Carry-Over factors transform each term of this matrix into the corresponding value for the far ends of each member, taking into account the different degrees of fixity than can be imposed. As an example, we can show the matrix (or table) of Carry-Over Factors corresponding to completely fixed far ends of members radiating from a specific node, with all its degrees of freedom blocked.

TABLE 2. CARRY-OVER FACTORS FOR MEMBERS WITH BOTH ENDS BUILT-IN

	Rotations □	Displacements X	Displacements Y
MOMENTS	1/2	-1	-1
X FORCES	1	-1	-1
Y FORCES	1	-1	-1

All these values follow the Cross conventions for signs (positive angles measured clockwise). If one multiplies these Carry-Over Factors, term by term, by the values of the Nodal Rigidity Matrix one obtains the following:

TABLE 3: PERIPHERAL (FAR END) RIGIDITY MATRIX WITH BOTH ENDS BUILT-IN

	Rotations □	Displacements X	Displacements Y
MOMENT M	2EI/L	(6EI/L2)sin□	-(6EI/L2)cos□
FORCE X	(6EI/L2)sin□	(AE/L)$\cos^2$□+ (12EI/L3)$\sin^2$□	-[AE/L - 12EI/L3]sin□cos□
FORCE Y	-(6EI/L2)cos□	-[AE/L - 12EI/L3]sin□cos□	- (AE/L)$\sin^2$□□(12EI/ L3)$\cos^2$□

If one wants to hinge the far end of a member going into a peripheral node, the following table applies (fixed-hinged):

TABLE 4. NODAL RIGIDITY MATRIX FOR MEMBERS WITH THE FAR END HINGED

	Rotation □	Displacement □x	Displacement □y
MOMENT M	3EI/L	($3EI/L^2$)sin□	-($3EI/L^2$)sin□
FORCE X	($3EI/L^2$)sin□	(AE/L)$\cos^2$□□□(3EI /L^3)$\sin^2$□	[AE/L- $3EI/L^3$] sin□cos□
FORCE Y	-($3EI/L^2$)sin□	[AE/L- $3EI/L^3$] sin□cos□	(AE/L)sen^2□□□(3EI /L^3)$\cos^2$□

Which can be obtained from the table for built-in ends applying the following modification factors (Cross-Style):

TABLE 5. MODIFICATION FACTORS FOR MEMBERS WITH THE FAR END HINGED

	Rotations □	Displacements X	Displacements Y
MOMENTS	3/4	1/4	1/4
X FORCES	1/2	1; 1/4	1; 1/4
Y FORCES	1/2	1; 1/4	1; 1/4

TABLE 6. CARRY-OVER FACTORS FOR MEMBERS WITH THE FAR END HINGED

	Rotations □	Displacements X	Displacements Y
MOMENTS	0	0	0
X FORCES	0	-1	-1
Y FORCES	0	-1	-1

As can be observed from these two examples, the old Cross Criteria of Carry-Over Factors and Modification Factors can be generalized to Matrix Structural analysis, which is not surprising. Similar tables can be obtained for other member ends conditions. In some special cases, like angled roller supports, instead of factors we will have to deal with Carry-Over Functions of angles.

Now let us show how graph theory can be applied to fill the local matrices and to assemble the Rigidity matrix of a Structure:

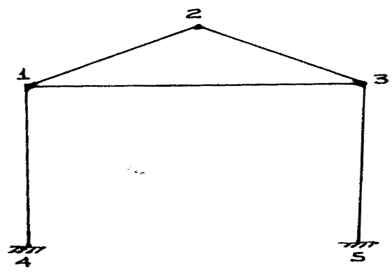

FIGURE 1
Gable Frame with Tie Beam

We will solve this example using the simplified matrix method procedure
We start with the corresponding connectivity matrix, including both the active (central) nodes (movable) and the passive (peripheral) nodes (fixed). In this case nodes 1; 2 and 3 are active; 4 and 5 passive.

TABLE 7. CONNECTIVITY MATRIX FOR THE FRAMED STRUCTURE OF FIG. 1 FILLED WITH MEMBER RIGIDITIES, EXCEPT FOR DIAGONAL CELLS

	Node 1	Node 2	Node 3	*Node 4*	*Node 5*
Node 1	ooo	**xxx**	**xxx**	**zzz**	ooo
Node 2	**xxx**	ooo	**xxx**	ooo	ooo
Node 3	**xxx**	**xxx**	ooo	ooo	**zzz**

This operative table has to be filled for every dimensional field: □-□□□□□□□x;□□y. X□□□□X□x; X□y. Y□□Y□x; Y□y with the corresponding rigidity coefficients.
Once this table has been filled, the SUMS OF ALL TERMS CONTAINED IN A ROW ARE PUT
INTO THE DIAGONAL CELLS, then, the passive nodes terms are suppressed from the table, as follows:
TABLE 8. OPERATIONAL TABLE TO COMPUTE THE DIAGONAL TERMS

	Node 1	Node 2	Node 3
Node 1	**+++**	**xxx**	**xxx**
Node 2	**xxx**	**+++**	**xxx**
Node 3	**xxx**	**xxx**	**+++**

TABLE 9. ALL NON-DIAGONAL TERMS ARE MULTIPLIED BY THE CORRESPONDING CARRY-OVER FACTORS

	Node 1	Node 2	Node 3
Node 1	**+++**	xxx	xxx
Node 2	xxx	**+++**	xxx
Node 3	xxx	xxx	**+++**

Once every dimensional field has been treated this way, a process of COPY & PASTE can perform the complete assemblage of the Rigidity Matrix:

TABLE 10. RIGIDITY MATRIX FOR THE STRUCTURE OF FIG. 1 (GABLE FRAME)

Variable	□	□	□	□x	□x	□x	□y	□y	□y
Node	1	2	3	1	2	3	1	2	3
M1	**++++**	Mo12	Mo13	**++++**	Mx12	Mx13	**++++**	My12	My13
M2	Mo21	**++++**	Mo23	Mx21	**++++**	Mx31	My21	++++	My23

Variable	□	□	□	□x	□x	□x	□y	□y	□y
M3	Mo31	Mo32	++++	Mx31	Mx32	++++	My31	My32	++++
X1	++++	Xo12	Xo13	++++	Xo12	Xo13	++++	Xy12	Xy13
X2	Xo21	++++	Xo23	Xx21	++++	Xo23	Xy21	++++	Xy21
X3	Xo31	Xo32	++++	Xx31	Xx32	++++	Xy31	Xy32	++++
Y1	++++	Yo12	Yo13	++++	Yx12	Yx13	++++	Yy13	Yy13
Y2	Yo21	++++	Yo23	Yx21	++++	Yx23	Yy21	++++	Yy23
Y3	Yo31	Yo32	++++	Yx31	Yx32	++++	Yy31	Yy32	++++

M,n represent Nodal Moments, Hn horizontal Nodal Forces and Yn Vertical Nodal Forces.

Now, we will show some examples of member properties when the end conditions change.

TABLE 11. RIGIDITY COEFFICIENTS FOR i END FIXED, j END ON ROLLER

	θz	Δx	Δy
Mz	$\frac{3EI}{L^2}$	$\frac{3EI}{L^2}\sin\alpha$	$-\frac{3EI}{L^2}\cos\alpha$
X	$\frac{3EI}{L^2}\sin\alpha$	$\frac{3EI}{L^3}\sin^2\alpha$	$-\frac{3EI}{L^3}\cos\alpha\sin\alpha$
Y	$-\frac{3EI}{L^2}\cos\alpha$	$-\frac{3EI}{L^3}\cos\alpha\sin\alpha$	$\frac{3EI}{L^3}\cos^2\alpha$

TABLE 12. RIGIDITY COEFFICIENTS FOR I END ON ROLLER, J END FIXED

	θz	Δx	Δy
Mz	0	0	0
X	0	$\frac{3EI}{L^3}\sin^2\alpha$	$-\frac{3EI}{L^3}\cos\alpha\sin\alpha$
Y	0	$-\frac{3EI}{L^3}\cos\alpha\sin\alpha$	$\frac{3EI}{L^3}\cos^2\alpha$

TABLE 13. RIGIDITY COEFFICIENTS FOR BOTH ENDS HINGED

	θz	Δx	Δy
Mz	0	0	0
X	0	$\frac{AE}{L}\cos^2\alpha$	$\frac{AE}{L}\cos\alpha\sin\alpha$
Y	0	$\frac{AE}{L}\cos\alpha\sin\alpha$	$\frac{AE}{L}\sin^2\alpha$

TABLE 14. RIGIDITY COEFFICIENTS FOR I END ON ROLLER, J END BUILT-IN

	θz	Δx	Δy
Mz	$\frac{3EI}{L^2}$	$\frac{3EI}{L^2}\sin\alpha$	$-\frac{3EI}{L^2}\cos\alpha$
X	$\frac{3EI}{L^2}\sin\alpha$	$\frac{3EI}{L^3}\sin^2\alpha$	$-\frac{3EI}{L^3}\cos\alpha\sin\alpha$
Y	$-\frac{3EI}{L^2}\cos\alpha$	$-\frac{3EI}{L^3}\cos\alpha\sin\alpha$	$\frac{3EI}{L^3}\cos^2\alpha$

TABLE 15. CARRY-OVER FACTORS, i END BUILT-IN; j END BUILT-IN.

	□	□x	□y
M	**1/2**	**-1**	**-1**
X	**1**	**-1**	**-1**
Y	**1**	**-1**	**-1**

TABLE 16 CARRY-OVER FACTORS i END BUILT-IN; j END HINGED. i END BUILT-IN; j END HINGED. BOTH ENDS HINGED.

	□	□x	□y
M	**0**	**0**	**0**
X	**0**	**-1**	**-1**
Y	**0**	**-1**	**-1**

TABLE 17. CANTILEVERS

Carry over factors for a Cantilever:	**All are ZERO for node to free end, 1 for the free end to node (the free end is a node).**

TABLE 18. CARRY-OVER FACTORS FOR ONE SINGLE TRANSVERSELY SLIDING BUILT-IN END (EITHER END)

	□	□x	□y
M	**1**	**-1**	**-1**
X	**1**	**-1**	**-1**
Y	**1**	**-1**	**-1**

TABLE 19. CARRY-OVER FACTORS FOR AN AXIALLY SLIDING BUILT-IN END. VALID FOR BOTH ENDS

	□	□x	□y
M	**1/2**	**-1**	**-1**
X	**1**	**-1**	**-1**
Y	**1**	**-1**	**-1**

These Tables can be prepared for different member end conditions and for members with variable sections, as is done in the Cross Method, the only difference being that instead of dealing only with moments and nodal angles we now deal with moments and forces and with nodal angles and nodal displacements. This allows the permanence of a single pattern for all rigidity matrices. Columns and nodes need to be suppressed only in some special cases, like having inclined supports and having partially fixed nodes attached directly to ground.

All this is done to preserve the similarities between the old methods (Cross, Slope-deflection) and the new Matrix Methods. We insisist that these procedures give preference to structural engineering teaching over computing methods. The idea is to have as few patterns to remember as possible, and to be able to solve classroom exercises with ordinary hand-held calculators or with widely available EXCEL™ or similar computer programs.
All this is documented in the references (1) Cubides, 2003 and (2) Vollalobos, 2004.

EXTENSION TO THREE DIMENSIONAL STRUCTURES :

At present we are initiating our efforts to find out if something similar to the indicated procedure can be applied to spatial structures. What follows is based on reference (3) Jiménez. 2004

It appears that a relatively simple procedure like the one shown above most probably will not be possible. What came out of Jiménez work is that the patterns of rigidity Matrices, the Cary-Over Factors and the Modification Factors can be applied in the same way. What is interesting, and this we have not found yet in textbooks about Matrix Analysis of Structures, is the tremendous complexity of the Spatial Rigidity Matrix, because this difficulty is circunvented by the systematic repetition of a triple Matrix Multiplication to take care of the orientation changes of the members, and forgetting about its internal structure. What follows are samples of some of the results obtained until now. The work proceeds with other students at present.

The basic dimensional structure of every rigidity coefficient in any Rigidity Matrix is always the same, it is composed of three terms, which can be identified in any of the tables already shown.

1) A non-dimensional configurational parameter which depends on the end conditions of the member, such as 2, 3, 4, 6, 12 For constant section members they are whole numbers, they may be fractional numbers if we deal with members with variable sections.

2) A dimensional term depending on the nature of the imposed deformation (angles, displacements) and the nature of the resultant effect, such as EI/L, AE/L, EI^3/L, EI^2/L etc.

3) A trigonometrical coefficient, depending on the orientation of the member (polar radius angle)

All this can be derived using Dimensional Analysis and these characteristics will always be present in all rigidity (Fourier Principle). So, nothing of this changes for spatial problems. The Basic Member Spatial Matrix has 36 terms, instead of the 9 shown for the plane case, because we have six possible generalized displacements and six generalized forces., as shown here:

TABLE 20.
SIMBOLIC SPATIAL RIGIDITY MATRIX

	□x	□y	□z	□X	□Y	□Z
M x	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}
M Y	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}
M	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}	C_{36}

z						
X	C_{41}	C_{42}	C_{43}	C_{44}	C_{45}	C_{46}
Y	C_{51}	C_{52}	C_{53}	C_{54}	C_{55}	C_{56}
Z	C_{61}	C_{62}	C_{63}	C_{64}	C_{65}	C_{66}

TABLE 21. LET US NOW SIMPLFY THE NOTATION. SAME TABLE 20, DIFFERENT SYMBOLS

I	II	IV	VII	XI	XVI
II	III	V	VIII	XII	XVII
IV	V	VI	IX	XIII	XVIII
VII	VIII	IX	X	XIV	XIX
XI	XII	XIII	XIV	XV	XX
XVI	XVII	XVIII	XIX	XX	XXI

TABLE 22. NOW WE LABEL THE DIRECTION COSINES LIKE THIS

a	b	c	0	0	0
d	e	f	0	0	0
g	h	i	0	0	0
0	0	0	a	b	c
0	0	0	d	e	f
0	0	0	g	h	i

If we now perform the triple multiplication of matrices to change coordinates from local to global, we get the following result for the Global Rigidity Matrix for the Spatial case, in symbolic form. The relationships are rather complex because of the interaction of terms from different positions.

TABLE 23. THREE-DIMENSIONAL RIGIDITY MATRIX

Relation	Cij	Components of the Rigidity Factor
M_X.□x	[11]	$I.a^2+2II.ad+2IV.ag+III.d^2+2V.dg+VI.g^2$
M_Y□x	21	I.ab+ III.de+VI.gh+II(db+ae)+IV(gb+ah)+V(ge+dh)
M_Z.□□□	31	I.ac+III.df+VI.gi+II(dc+af)+IV(gc+ai)+V(gf+di)
X. □ x	41	$VII.a^2+XII.d^2+XVIII.g^2+XI.da+XVI.ga+VIII.ad+XVII.gd+IX.ag$ $+XIII.dg$
Y.□□ x	51	VII.ba+XI.ea+XVI.ha+VIII.bd+XII.ed+XVII.hd+IX.bg+XIII.eg

		+XVIII.hg
Z.□□ x	61	VII.ca+XI.fa+XVI.ia+VIII.cd+XII.fd+XVII.id+IX.cg+XIII.fg+XVIII.ig
MY.□□ Y	[22]	I.b^2+III.e^2+VI.h^2+2II.be+2IV.hb+2V.eh
MZ.□□ Y	32	I.cb+III.fe+VI.ih+II(fb+ce)+IV(ib+ch)+V(ie+fh)
X.□□Y	42	VII.ab+XI.db+XVI.gb+VIII.ae+XII.de+XVII.ge+IX.ah+XIII.dh+XVIII.gh
Y.□□ Y	52	VII.b^2+XII.e^2+XVIII.h^2+XI.eb+XVI.hb+VIII.be+XVII.he+IX.bh+XIII.eh
Z.□□ Y	62	VII.cb+XI.fb+XVI.ib+VIII.ce+XII.fe+XVII.ei+IX.ch+XIII.fh+XVIII.hi
MZ.□□Z	[33]	I.c^2+III.f^2+VI.i^2+2II.fc+2IV.ic+2V.if
X.□□ Z	43	VII.ac+XI.dc+XVI.gc+VIII.af+XII.df+XVIIgf+IX.ai+XIII.di+XVIII.gi
Y.□□ Z	53	VII.bc+XI.ec+XVI.hc+VIII.bf+XII.ef+XVI.hf+IX.bi+XIII.ei+XVIII.hi
Z. □Z	63	VII.c^2+XI.fc+XVI.ic+VIII.cf+XII.f^2+XVII.if+IX.ci+XIII.fi+XVIII.i^2
X.□ X	[44]	X.a^2+XV.d^2+XXI.g^2+2XIV.ad+2XIX.ag+2XX.dg
Y. □ X	54	X.ba+XIV.ea+XIX.ha+XIV.bd+XV.ed+XX.hd+XIX.bg+XX.eg+XXI.hg
Z. .□ X	64	X.ca+XV.fd+XXI.ig+XIV(fa+cd)+XIX(ia+cg)+XX(id+fg)
Y.□ Y	[55]	Xb^2+XV.e^2+XXI.h^2+XIV.eb+XIX.hb+XIV.be+XX.he+XIX.bh+XX.eh.
Z. □ Y	65	X.cb+XV.fe+XXI.ih+XIV(fb+ce)+XIX(bi+ch)+XX(ie+fh)
Z. □ Z	[66]	X.c^2+XIV.cf+XIX.ic+XIV.cf+XV.f^2+XX.if+XIX.ci+XX.fi+XXI.i^2

It can also be said that the basic structure of the resultant rigidity coefficients, with three definite components, stays. As it was expected from the Dimensional Analysis rule.

Notice: Only the lower triangular part of the Rigity matrix is shown in this Table. The remaining terms can be deduced by symmetry.

TABLE 24. THE RESULTING TERMS OF TABLE 23 IN DIMENSIONAL FORM, FOR A MEMBER WITH BOTH ENDS RIGIDLY ATTACHED TO THE NODES

	□X	□Y	□Z	□X	□Y	□Z
MX	$\frac{GJ}{L}$	$\frac{4EI}{L}$	$\frac{4EI}{L}$	$\frac{6EI}{L^2}$	$\frac{6EI}{L^2}$	$\frac{6EI}{L^2}$
MY	$\frac{4EI}{L}$	$\frac{GJ}{L}$	$\frac{4EI}{L}$	$\frac{6EI}{L^2}$	$\frac{6EI}{L^2}$	$\frac{6EI}{L^2}$
MZ	$\frac{4EI}{L}$	$\frac{4EI}{L}$	$\frac{GJ}{L}$	$\frac{6EI}{L^2}$	$\frac{6EI}{L^2}$	$\frac{6EI}{L^2}$
X	$\frac{6EI}{L^2}$	$\frac{6EI}{L^2}$	$\frac{6EI}{L^2}$	$\frac{AE}{L}$	$\frac{12EI}{L^3}$	$\frac{12EI}{L^3}$
Y	$\frac{6EI}{L^2}$	$\frac{6EI}{L^2}$	$\frac{6EI}{L^2}$	$\frac{12EI}{L^3}$	$\frac{AE}{L}$	$\frac{12EI}{L^3}$
Z	$\frac{6EI}{L^2}$	$\frac{6EI}{L^2}$	$\frac{6EI}{L^2}$	$\frac{12EI}{L^3}$	$\frac{12EI}{L^3}$	$\frac{AE}{L}$

If we want to apply table 24 to a member with the i end rigidly attached and the j end hinged, we can apply the modification factors indicated in the following table, (25), in a similar fashion as in the Cross Method (multiplying cell by cell). NOTE: *Table 25 is not a matrix.*

TABLE 25. MODIFICATION FACTORS

	□X	□Y	□Z	□X	□Y	□Z
MX	1	3/4	3/4	2/4	2/4	2/4
MY	3/4	1	3/4	2/4	2/4	2/4
MZ	3/4	3/4	1	2/4	2/4	2/4
X	2/4	2/4	2/4	1	1/4	1/4
Y	2/4	2/4	2/4	1/4	1	1/4
Z	2/4	2/4	2/4	1/4	1/4	1

We are not giving more tables here, the ones shown already stress the relationships between the Cross Method and the Matrix methods.
It is worthwhile to notice that the student does not perform Matrix multiplications, except when he inverts the Rigidity Matrix to obtain the Flexibility Matrix and after that multiplies the Flexibility Matrix and the Load Vector.

The Rigidity Matrices resulting from this method are not optimal from the point of view of computational precision. The intended purpose of these procedures is to facilitate the comprehension of the Matix Method and to relate it to the Classical Methods, which are still taught or referred to in the texbooks.
EXCEL™ in its standard form does not accept floating point mantissas with more than 15 or 16 numbers. Its advantage is that it comes almost with every laptop. Some good Hand held programmable Calculators could also be used in this method, if one accepts their more cumbersome ways of entering data.
As a last example, here is the Table of Carry-Over Factors for the "standard" double buit-in member. One can see that the Carry-Over concept can be extended to Space Frames and not only to moments, but also to shears and axial forces.

TABLE 26. CARRY-OVER FACTORS. MEMBERS WITH BOTH ENDS BUILT-IN

	□X	□Y	□Z	□X	□Y	□Z
MX	1	½	½	−1	−1	−1
MY	½	1	½	−1	−1	−1
MZ	½	½	1	−1	−1	−1
X	1	1	1	−1	−1	−1
Y	1	1	1	−1	−1	−1
Z	1	1	1	−1	−1	−1

CONCLUSIONS: *This simplified Matrix Method has been received very well by the students, because they have to learn only a few basic rules, and it can be performed manually. They may use their own hand calculators or their own laptops. Excel™ is already familiar to them, so they tend to use its language as a part of their thinking processes.*
In today's world it seems that it does not make more sense to force them to use algorithmic languages. One can no longer compete with the commercial packages, so the understanding of the structural principles and the perception of unity between diverse classical methods is nowadays more important for teaching purposes than the ability to program.

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