

Generalized Fuzzy Payback Period

Javier Ríos Valledepaz

Departamento de Finanzas

Universidad Metropolitana

Caracas-Venezuela

jrios@unimet.edu.ve

ABSTRACT

The Fuzzy Payback Period (FPP) is an extension for fuzzy cash flows of the concept of payback period for real cash flows but this criterion is not consistent with the criterion of fuzzy net present value. This paper defines a variant of FPP, the generalized fuzzy payback period (GFPP), that allows establishing a criterion of selection for independent projects with fuzzy cash flows, consistent with the criterion of the fuzzy net present value

Keywords: fuzzy cash flows, fuzzy payback period, fuzzy net present value

1. INTRODUCTION

The construction of the investment project cash flow implies the estimation of the future income and outcome, which depends on other parameters like inflation or interest rates.

These variables are generally uncertain and if it isn't exist reliable probabilistic information, the statistical methods are not appropriate. When the uncertainty is reflected of the lack of information, it is generally feasible to establish a rank of values possible, that it can be represented by means of trapezoidal or triangular fuzzy numbers, which can be interpreted like families of intervals with different levels from possibility (Bodjanova, 2002). On the other hand, the Fuzzy Arithmetic is conceptually simple and easy to interpret (Buckley, 1987), although lacks fundamental properties like the distributive property or the existence of inverse elements (Ríos, 1999). However, the operations with trapezoidal numbers can be transferred of natural way to a spreadsheet, for this reason the apparent complexity of the calculations is eliminated.

2. FUZZY PAYBACK PERIOD

The Fuzzy Payback Period (FPP) is an extension for fuzzy cash flows of the concept of payback period for real cash flows. It is defined as the first period in which the accumulated balance of the cash flow, for a rate reinvestment, takes positive value. When the fuzzy cash flow is conventional then this number exists, but with non-conventional flows it cannot exist. In this case, average fuzzy payback period (AVPP) can be defined like the first period in which the value average of the accumulated balance of the fuzzy cash flow changes of sign. In both cases, the payback period can be compared against an acceptable maximum payback period like selection criterion. Nevertheless, as the real case, this criterion is not consistent with the criterion of fuzzy present value net (Chiu y Park, 1994), because this one is a yield measurement, whereas the payback period is a liquidity measurement (Ertugrul, 1998).

Definition 2.1

If $S = [S_1, S_2, S_3, S_4]$ is the trapezoidal rate of reinvestment for a trapezoidal cash flow $F_j = [F_{j1}, F_{j2}, F_{j3}, F_{j4}]$ with $j = 0, 1, \dots, n$, then G_j is the accumulated balance in period j , when $G_{j+1} = G_j * (1+S) \oplus F_{j+1}$ with $G_0 = F_0$.

If m exists so that $G_m > 0$ and $G_{m-1} < 0$ then the payback period of F_j is m . If it does not exist, average payback period can be defined. In both cases, payback period can be compared against a period maximum acceptable for effects of acceptance or rejection

Generalized Fuzzy Payback Period

The generalized fuzzy payback period (GFPP) is the absolute value of the initial cash flow, multiplied by the inverse one of the fuzzy equivalent annual value of the fuzzy flow from the first period to the last one. This index allows to establish a criterion of selection for independent projects with fuzzy cash flows, consistent with the criterion of the fuzzy net present value (FNPV).

Definition 2.2

If F_j is a trapezoidal cash flow in period j , $F_j = [F_{j1}, F_{j2}, F_{j3}, F_{j4}]$ $j = 0, 1, \dots, n$, and K is a rate of discount per period, $K = [K_1, K_2, K_3, K_4]$, $K_1 > 0$ then $FPV = [P_1, P_2, P_3, P_4]$ with $P_i = \sum_{j=1..n} F_{ji} * (1+K_i)^{-j}$, $i = 1, 2, 3, 4$ is the fuzzy present value of the cash flow F_j .

The fuzzy equivalent annual value is

$$FEAV = FPV * \Pi^{-1}$$

and the fuzzy equivalent annual factor is

$$\Pi = [\Pi_4, \Pi_3, \Pi_2, \Pi_1], \Pi_i = (1 - (1+K_i)^{-n}) / K_i,$$

The generalized fuzzy payback period GFPP is defined as the trapezoidal number

$$GFPP = -F_0 * FEAV^{-1}$$

with $FEAV > 0$ and $F_0 < 0$.

Properties

If F_j is a fuzzy cash flow with $F_0 < 0$ and fuzzy equivalent annual value $FEAV > 0$, then generalized fuzzy payback period $GFPP < \Pi$ implies that the fuzzy net present value $FNPV > 0$ and $GFPP > \Pi$ implies that the fuzzy net present value $FNPV < 0$.

Proof

If $FPV = [P_1, P_2, P_3, P_4]$ then $FEAV$ is

$$FEAV = [P_1 \Pi_1^{-1}, P_2 \Pi_2^{-1}, P_3 \Pi_3^{-1}, P_4 \Pi_4^{-1}]$$

and $GFPP = [-F_0 P_4^{-1} \Pi_4, -F_0 P_3^{-1} \Pi_3, -F_0 P_2^{-1} \Pi_2, -F_0 P_1^{-1} \Pi_1]$

If $GFPP < \Pi$ then $-F_0 P_1^{-1} \Pi_1 < \Pi_4$ hence $-F_0 < P_1 \Pi_1^{-1} \Pi_4 < P_1$, that is, $F_0 + P_1 > 0$ therefore $FNPV = F_0 + FPV > 0$

If $GFPP > \Pi$ then $-F_0 P_4^{-1} \Pi_4 > \Pi_1$ hence $-F_0 > P_4 \Pi_4^{-1} \Pi_1 > P_4$, that is, $F_0 + P_4 < 0$ therefore $FNPV = F_0 + FPV < 0$.

There is a graphic example in the appendix that helps interpret the concepts of generalized fuzzy payback period

Since these models use fuzzy information, the obtained result is a trapezoidal number (Chalco, 2009). The level of possibility allows to restrict the result and can be used like optimism level (Kim y Park, 1990) and, in addition, numerous methods of ranking fuzzy numbers can be applied (Chen, 1985; Liou, 1992).

3. CONCLUSIONS

The fuzzy payback period (FPP) and the average fuzzy payback period (AFPP) are alternative indices when a fuzzy liquidity measurement is desired. Nevertheless, these indices are not consistent with the fuzzy present value net. On the contrary, the generalized fuzzy payback period (GFPP) is a consistent variant with the criterion of the fuzzy net present value (FNPV) that allows establishing a criterion of selection for independent projects with fuzzy cash flows.

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Appendix

Generalized Fuzzy Payback Period

	K1	K2	K3	K4				
	4,8%	6,5%	7,2%	8,2%				
j	Fj1	Fj2	Fj3	Fj4	FPVj1	FPVj2	FPVj3	FPVj4
0	-1020	-1000	-990	-970	-1020	-1000	-990	-970
1	213	256	266	271	197	238	250	259
2	349	386	391	393	298	336	340	342
3	302	308	331	370	238	250	269	300
4	210	257	257	307	153	195	195	233
5	302	353	379	395	203	249	267	279
6	152	202	202	239	95	133	133	157
7	91	131	151	202	52	80	93	124
8	111	115	140	163	59	66	80	93
9	58	116	144	159	28	62	77	85
10	37	88	116	156	17	44	58	78
	Π1	Π2	Π3	Π4	FPV	FEAV	GFPP	
	7,807	7,1958	6,9577	6,6354	1340	1654	1761	1950
					172	230	253	294
					3,3	3,9108	4,3507	5,944

